Contract Theory: A New Frontier for AGT

Part I: Classic Theory

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Plan

- Part I (Inbal): Classic Theory
  - Model
  - Optimal Contracts
  - Key Results

- Break (5-10 mins)

- Part II (Paul): Modern Approaches
  - Robustness
  - Approximation
  - Computational Complexity

*We thank Tim Roughgarden for feedback on an early version and Gabriel Carroll for helpful conversations; any mistakes are our own
1. What is a Contract?
An Old Idea

Les Mines de Bruoux, dug circa 1885

All of the workers were paid by the day, except for the leading miner who was paid according to the distance he dug. The most qualified of them could progress up to 1 meter in a day. In 1885, the workers earned between 0.25F and 0.40F per hour.

At the time they used carbide lamps, also called acetylene lamps.

The cast iron pipes were used to expel the
Purpose of Contracts

• Contracts **align interests** to enable exploiting gains from cooperation

• “What are the common wages of labour, depends everywhere upon the contract usually made between those two parties, **whose interests are not the same.**” [Adam Smith 1776]
Classic Contract Theory

“Modern economies are held together by innumerable contracts”

[2016 Nobel Prize Announcement]

Laureates Oliver Hart and Bengt Holmström
Classic Applications

• Employment contracts
• Venture capital (VC) investment contracts
• Insurance contracts
• Freelance (e.g. book) contracts
• Government procurement contracts
• ...

→ Contracts are indeed everywhere
New Applications

Classic applications are moving **online** and/or increasing in **complexity**

- Crowdsourcing platforms
- Platforms for hiring freelancers
- Online marketing and affiliation
- Complex supply chains
- Pay-for-performance medicare

→ **Algorithmic** approach becoming more relevant
Basic Contract Setting [Holmström’79]

• 2 players: principal and agent
• Familiar ingredients: private information and incentives
• Let’s see an example...
Example

• Website owner (principal) hires marketing agent to attract visitors

• Two defining features:
  1. Agent’s actions are hidden - “moral hazard”
  2. Principal never charges (only pays) agent - “limited liability”
Moral Hazard

“Well then, says I, what’s the use of you learning to do right when it’s troublesome to do right and ain’t no trouble to do wrong, and the wages is just the same?”

Mark Twain, *Adventures of Huckleberry Finn*
Limited Liability

**Typical example:** an entrepreneur and a VC

- The entrepreneur builds the company
- The VC diversifies the risks and has deep pockets
Principal offers agent a contract (parties have symmetric info)

Agent accepts (or refuses)

Agent takes costly, hidden action

Action’s outcome rewards the principal

Principal pays agent according to contract
2. Connection to AGT
Relation to Other Incentive Problems [Salanie]

<table>
<thead>
<tr>
<th>Private information is hidden type</th>
<th>Uninformed player has the initiative</th>
<th>Informed player has the initiative</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mechanism design (screening)</td>
<td>Signaling (persuasion)</td>
<td></td>
</tr>
</tbody>
</table>

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<td></td>
<td>-</td>
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</table>
New Frontier

• Economics and computation – lively interaction over past 2 decades
• Especially true for mechanism design and signaling

Can we recreate the success stories of AGT in the context of contracts?

• Are insights from CS useful for contracts? Is contract theory useful for AGT applications? In Part II: A preliminary YES to both
Already Building Momentum

• Pioneering works:
  • Combinatorial agency [Babaioff Feldman and Nisan’12,...]
  • Contract complexity [Babaioff and Winter’14,...]
  • Incentivizing exploration [Frazier Kempe Kleinberg and Kleinberg’14]
  • Robustness [Carroll’15,...]
  • Adaptive design [Ho Slivkins and Vaughan’16,...]

• Recent works:
  • Delegated search [Kleinberg and Kleinberg’18,...]
  • Information acquisition [Azar and Micali’18,...]
  • Succinct models [Dütting Roughgarden and T.-C.’19b,...]

• EC’19 papers:
  • [Kleinberg and Raghavan’19, Lavi and Shamash’19, Dütting Roughgarden and T.-C.’19a]
The Algorithmic Lens

• Offers a language to discuss complexity
• Has popularized the use of approximation guarantees when optimal solutions are inappropriate
• Puts forth alternatives to average-case / Bayesian analysis that emphasize robust solutions to economic design problems

More on this in Part II
But first, let’s cover the basics
3. Formal Model
Contract Setting

• Parameters $n, m$

• Agent has actions $a_1, \ldots, a_n$
  • with costs $0 = c_1 \leq \cdots \leq c_n$ (can always choose action with 0 cost)

• Principal has rewards $0 \leq r_1 \leq \cdots \leq r_m$

• Action $a_i$ induces distribution $F_i$ over rewards (“technology”)
  • with expectation $R_i$
  • Assumption: $R_1 \leq \cdots \leq R_n$

• Contract = vector of transfers $\vec{t} = (t_1, \ldots, t_m) \geq 0$
Example

<table>
<thead>
<tr>
<th>Contract:</th>
<th>$t_1 = 0$</th>
<th>$t_2 = 1$</th>
<th>$t_3 = 2$</th>
<th>$t_4 = 5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>No visitor $r_1 = 0$</td>
<td>General visitor $r_2 = 3$</td>
<td>Targeted visitor $r_3 = 7$</td>
<td>Both visitors $r_4 = 10$</td>
<td></td>
</tr>
<tr>
<td>Low effort $c_1 = 0$</td>
<td>0.72</td>
<td>0.18</td>
<td>0.08</td>
<td>0.02 $R_1 = 1.3$</td>
</tr>
<tr>
<td>Medium effort $c_2 = 1$</td>
<td>0.12</td>
<td>0.48</td>
<td>0.08</td>
<td>0.32 $R_2 = 5.2$</td>
</tr>
<tr>
<td>High effort $c_3 = 2$</td>
<td>0</td>
<td>0.4</td>
<td>0</td>
<td>0.6 $R_3 = 7.2$</td>
</tr>
</tbody>
</table>
Expected Utilities

Fix action $a_i$.

Agent

- $\mathbb{E}[\text{utility}] = \text{expected transfer } \sum_{j \in [m]} F_{i,j} t_j \text{ minus cost } c_i$

Principal

- $\mathbb{E}[\text{payoff}] = \text{expected reward } R_i \text{ minus expected transfer } \sum_j F_{i,j} t_j$

Utilities sum up to $R_i - c_i$, action $a_i$’s expected welfare

Contract setting:
- $n$ actions $\{a_i\}$, costs $\{c_i\}$
- $m$ rewards $\{r_j\}$
- $n \times m$ matrix $F$ of distributions with expectations $\{R_i\}$

Payoff ≠ payment/transfer
Example: Agent’s Perspective

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<td>0.4</td>
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Expected transfers: (0.44, 2.24, 3.4) for (low, medium, high)
Example: Agent’s Perspective

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Expected transfers: (0.44, 2.24, 3.4) for (low, medium, high)
Example: Principal’s Perspective

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\( R_3 - \text{expected transfer} = 7.2 - 3.4 = 3.8 \)
A Remark on Risk Averseness

• Recall 2nd defining feature: agent has limited liability ($\bar{t} \geq 0$) [Innes’90]

• Popular alternative to risk-averseness
  • Utility from transfer $t_j$ is $u(t_j)$ where $u$ strictly concave

• Both assumptions justify why the agent enters the contract
  • Rather than “buying the project” and being her own boss
A Remark on Tie-Breaking

• **Standard assumption**: If the agent is *indifferent* among actions, he chooses the one that *maximizes the principal’s* expected payoff
4. Computing Optimal Contracts
Contract Design

**Goal:** Design contract that maximizes principal’s payoff

Optimization s.t. incentive compatibility (IC) constraints:

- Maximize $\mathbb{E}[\text{payoff}]$ from action $a_i$
- Subject to $a_i$ maximizing $\mathbb{E}[\text{utility}]$ for agent

Related Problems: Implementability of action $a_i$; min pay for action $a_i$

Can all be solved using LPs!
First-Best Benchmark

• **First-best** = solution *ignoring* IC constraints

• What principal could extract if actions *weren’t hidden*
  • I.e., if could pick action and pay its cost

\[
\text{First-best} = \max_i \{R_i - c_i\}
\]

• **OPT ≠ first-best** due to IC constraints
Implementability Problem

Given: Contract setting; action $a_i$

Determine: Is $a_i$ implementable (exists contract $\tilde{t}$ for which $a_i$ is IC)

LP duality gives a simple characterization!

Proposition: Action $a_i$ is implementable (up to tie-breaking) $\iff$ no convex combination of the other actions has same distribution over rewards at lower cost
Implementability LP

\( a_i \) implementable \( \iff \) LP feasible

\( m \) variables \( \{ t_j \} \) (transfers); \( n - 1 \) IC constraints

minimize 0

s.t. \( \sum_j F_{i,j} t_j - c_i \geq \sum_j F_{i',j} t_j - c_{i'} \quad \forall i' \neq i \) (IC)

Agent’s expected utility from \( a_i \) given contract \( t \)

\( t_j \geq 0 \) (LL)
Dual* for Action $a_i$

Primal infeasible $\iff \exists$ feasible dual solution with objective $> 0$

$n - 1$ variables $\{\lambda_{i'}\}$ (weights); $m$ constraints

Combined cost

$$\text{maximize } c_i - \sum_{i' \neq i} \lambda_{i'} c_{i'}$$

Convex combination of actions

$$\text{s.t. } \sum_{i' \neq i} \lambda_{i'} F_{i',j} \leq F_{i,j} \ \forall j \in [m]$$

$$\lambda_{i'} \geq 0; \ \sum_{i' \neq i} \lambda_{i'} = 1$$
Min Pay Problem

• Find minimum total transfer of a contract implementing action $a_i$
• Same LP with updated objective:

$$\text{minimize} \quad \sum_j F_{i,j} t_j$$

s.t. \quad \sum_j F_{i,j} t_j - c_i \geq \sum_j F_{i',j} t_j - c_{i'} \quad \forall i' \neq i \quad \text{(IC)}$$

\[ t_j \geq 0 \quad \text{(LL)} \]
Optimal Contract Problem

Key observation:
• Can compute optimal contract by solving $n$ LPs, one per action

Run-time per LP:
• Polynomial in $n - 1$ (constraints), $m$ (variables)

Corollary:
• ∃ optimal contract with $\leq n - 1$ nonzero transfers
Criticism of LP-Based Approach

“More normative than positive”:

1. Requires perfect knowledge of distribution matrix $F$

2. What if polytime in $n, m$ is too slow?
   • Recall example: $m$ is exponential in number of visitor types to website

3. The contract that comes out of the LP may seem arbitrary
5. Structure of Optimal Contracts
Let $\pi > p$

<table>
<thead>
<tr>
<th>Action</th>
<th>“Failure” reward $r_1$ $\Vect{\text{bad}}$</th>
<th>“Success” reward $r_2 &gt; r_1$ $\ Vect{\text{good}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>“Shirking”</td>
<td>1 − $p$</td>
<td>$p$</td>
</tr>
<tr>
<td>cost = 0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>“Working”</td>
<td>1 − $\pi$</td>
<td>$\pi$</td>
</tr>
<tr>
<td>cost = $c &gt; 0$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Optimal Contract for $n = m = 2$

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
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<tbody>
<tr>
<td>1 - $p$</td>
<td>$p$</td>
</tr>
<tr>
<td>1 - $\pi$</td>
<td>$\pi$</td>
</tr>
</tbody>
</table>

Q: What does the optimal contract look like?

• Principal can always extract $R_1 = (1 - p)r_1 + pr_2$

→ Question interesting when optimal contract incentivizes work

• In this case:

$$\text{first-best} = R_2 - c = (1 - \pi)r_1 + \pi r_2 - c$$
Optimal Contract for $n = m = 2$

- **Notation**: Contract pays $t_1$ for failure, $t_2$ for success
- **IC constraint** for working is:
  \[ t_1(1 - \pi) + t_2\pi - c \geq t_1(1 - p) + t_2p \]
  \[ \iff (\pi - p)(t_2 - t_1) \geq c \text{ (*)&} \]

- (*&) binds at the optimal contract

→ Optimal contract is: $t_1 = 0$; $t_2 = \frac{c}{\pi - p}$

→ Principal extracts: $R_2 - c \left( \frac{\pi}{\pi - p} \right)$

Compare to first-best = $R_2 - c$
Optimal Contract for $n = m = 2$

Q: Structural properties of the optimal contract $t_1 = 0$; $t_2 = \frac{c}{\pi - p}$?

- **Monotonicity** property = transfer increases w/ reward
  - Generalizes to any $n$ as long as $m = 2$
- As $\pi, p$ draw closer, harder to distinguish work from shirk, so $t_2$ grows
Optimal Contract for 2 Actions, $m$ Rewards

- $n = 2, m > 2$

- **Recall**: There’s an optimal contract with $n - 1 = 1$ nonzero transfers
Optimal Contract for $n = 2, m > 2$

Q: Which reward $r_j$ gets the nonzero transfer $t_j$ in optimal contract?

• Binding IC constraint for working is $t_j F_{2,j} - c = t_j F_{1,j}$

$\rightarrow$ Optimal contract is $t_j = \frac{c}{F_{2,j} - F_{1,j}}$; principal extracts $R_2 - c \frac{F_{2,j}}{F_{2,j} - F_{1,j}}$
Optimal Contract for $n = 2, m > 2$

• Principal extracts $R_2 - c \left( \frac{1}{1 - \frac{F_{1,j}}{F_{2,j}}} \right)$

→ To maximize over all $j$, choose $j^*$ that minimizes $\frac{F_{1,j}}{F_{2,j}}$

• $\frac{F_{1,j}}{F_{2,j}}$ is called the likelihood ratio of actions $a_1, a_2$
  • Numerator (denominator) is likelihood of shirk (work) given reward $r_j$

**Takeaway:** Optimal contract pays for reward with min likelihood ratio
Optimal Contract for $n = 2, m > 2$

Recap

Q: Which reward $r_j$ gets the nonzero transfer $t_j$ in optimal contract?

A: Pay for $r_j$ with min likelihood ratio $\frac{F_{1,j}}{F_{2,j}}$

Statistical inference intuition (holds for general $n$):

Principal is inferring agent’s action from the reward

→ Pays more for rewards from which can infer agent is working
An Extreme Example

• Assume reward $r_j^*$ has nonzero probability $\epsilon$ only if agent works
  • I.e. if $r_j^*$ occurs, “gives away” agent’s action

• Optimal contract has single nonzero transfer $t_j^* = \frac{c}{\epsilon}$

• The good: Principal extracts first-best $= R_2 - c$

• The bad: Contract non-monotone
  • (Recall: monotone = transfer increases with reward)
Example with \( n > 2 \)

Optimal contract incentivizes action \( a_3 \)

<table>
<thead>
<tr>
<th>Contract:</th>
<th>( t_1 = 0 )</th>
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<th>( t_3 \approx .15 )</th>
<th>( t_4 \approx 3.9 )</th>
<th>( t_5 \approx 2 )</th>
<th>( t_6 = 0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r_1 = 1 )</td>
<td>3/8</td>
<td>3/8</td>
<td>2/8</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( c_1 = 0 )</td>
<td>3/8</td>
<td>3/8</td>
<td>2/8</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( c_2 = 1 )</td>
<td>0</td>
<td>3/8</td>
<td>3/8</td>
<td>2/8</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( c_3 = 2 )</td>
<td>0</td>
<td>0</td>
<td>3/8</td>
<td>3/8</td>
<td>2/8</td>
<td>0</td>
</tr>
<tr>
<td>( c_4 = 2.2 )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>3/8</td>
<td>3/8</td>
<td>2/8</td>
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Contract:

\( t_1 = 0 \)
\( t_2 = 0 \)
\( t_3 \approx .15 \)
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\( t_5 \approx 2 \)
\( t_6 = 0 \)
Recap

Role of rewards in the model is two-fold:
1. Represent *surplus* to be shared
2. *Signal* to principal the agent’s action

- The optimal contract is *shaped* by (2)
- Can be *mismatched* with (1)
6. Results on Monotonicity and Informativeness
Regularity Conditions [Mirrlees’99]

Q: Natural conditions for optimal contract monotonicity?

For \( n = 2 \) actions, \( m \)th reward must have min likelihood ratio

**Definition:** A contract setting satisfies MLRP (monotone likelihood ratio property) if

\[
\forall \text{ actions } a_i, a_i', i < i': \quad \frac{F_{i,j}}{F_{i',j}} \text{ decreasing in } j
\]

**Intuition:** The higher the reward, the more likely the higher-cost action

**Note:** MLRP implies FOSD (\( F_{i'} \) first-order stochastically dominates \( F_i \))
Regularity Conditions [MIRRLEES’99]

- **MLRP insufficient** for monotonicity with $n > 2$ (recall example)
- Sufficient with “CDF Property” or if actions have increasing welfare
  - “CDFP really has no clear economic interpretation, and its validity is much more doubtful than that of MLRP” [Salanie’05]

<table>
<thead>
<tr>
<th></th>
<th>$r_1 = 1$</th>
<th>$r_1 = 1.1$</th>
<th>$r_1 = 4.9$</th>
<th>$r_1 = 5$</th>
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Informativeness [Grossman-Hart’83]

Fix $n$ actions, $m \times m$ stochastic matrix $\Pi$

Consider 2 contract settings $(F, r), (F', r')$ s.t. $\forall$ action $a_i$:

- $R_i' = R_i$
- $F_i'$ obtained from $F_i$ as follows: Draw reward-index $j'$ by drawing $j$ from $F_i$, then drawing from $j$th column of $\Pi$

$\rightarrow$ Settings have same expected rewards but $F'$ is a coarsening of $F$

Proposition: Min pay for action $a_i$ is higher in coarser setting
Informativeness [Holmstrom’79]

Suppose principal can observe additional signals indicating action, e.g., a report from agent’s direct supervisor

Statistical model connection:
- Action = underlying parameter
- Reward + report = observed data

Given reward, does report give further info on action? If so – use it!

Sufficient statistic theorem: The principal should condition transfers on a sufficient statistic for all available signals
Recap

• Classic lit has made headway in making sense of optimal contracts
• E.g. through statistical inference connections

Limitations:
• Conditions like actions having increasing welfare are too strong
• “Coarsening” relation is a very partial order on contract settings
A Way Forward: Simple Contracts

• Linear contracts: Determined by parameter $\alpha \in [0,1]$
  • For reward $r_j$ the principal pays the agent $\alpha r_j$
  • Generalization to affine: $\alpha r_j + \alpha_0$

• Agent’s expected utility from action $a_i$ is $\alpha R_i - c_i$
• Principal’s expected payoff is $(1 - \alpha)R_i$

Notice: No dependence on details of distribution!
7. Model Extensions & Summary
Extensions

1. Continuum of actions: Studied in particular with 2 rewards [Mirrlees’99]

2. Continuum of rewards: Functional analysis [Page’87]

3. Multiple agents: Teamwork, free-riding [Holmstrom’82]

4. Multiple principals: Agent’s success in a project benefits 2 principals [Bernheim-Whinston’86]

5. Multitasking: Actions can be substitutes or complements for agent [Holmstrom-Milgrom’91]

6. Adverse selection: Agents also have hidden types [E.g., Chiappori et al.’94]
Dynamics

1. **Multiple time periods**, agent takes action at each period
   - In this model [Holmstrom and Milgrom’87] give first **robustness** explanation for real-life contracts taking a **simple, often linear** form

2. **Renegotiation** after action is taken
   - May prevent implementing costly actions
Incomplete Contracts

Famous example from 1920s [Klein et al.’78]:
• Contract between GM and car-part manufacturer
• GM committed; manufacturer kept costs high ("held up" GM)

Problem caused by incomplete contract setting:
• Players can make specific investments
• Not all appear in contract due to transaction costs [Coase’37]
→ Leads to underinvestment; here renegotiation can be socially useful
Recap of Part I

Contracts incentivize someone to do something for us although we get the rewards and they incur the cost

• A model with familiar components of private info, incentives that “fit together” in fundamentally different way than auctions

• Two defining features: (1) Hidden actions (2) Limited agent liability
Recap of Part I: Main Results

• **Implementability, Min Pay and Optimal Contract** all solvable with LPs in poly($n, m$) runtime if distributions known

<table>
<thead>
<tr>
<th>Optimal Contract</th>
<th>2 rewards</th>
<th>$m$ rewards</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>2 actions</strong></td>
<td>Monotonicity &amp; Pay for reward with min likelihood ratio</td>
<td>Pay for reward with min likelihood ratio</td>
</tr>
<tr>
<td><strong>$n$ actions</strong></td>
<td>Monotonicity</td>
<td>Strong assumptions needed</td>
</tr>
</tbody>
</table>
Resources


*See Appendices of [Dütting Roughgarden and T.-C.’19a] for more details on many of the basics covered in this tutorial
*For tutorial bibliography see tutorial website
Questions?

- After the break: Algorithmic aspects