Beyond Worst-Case Analysis in Algorithmic Game Theory

Inbal Talgam-Cohen, Technion CS

Games, Optimization & Optimism: Workshop in Honor of Uri Feige
Weizmann Institute, January 2020
Uri as an advisor

Q1: What did you appreciate most about Uri as an advisor?

Q2: What did you learn from him that has proved most meaningful over the years?
Uri as an advisor

• “Uri has scientific x-ray eyes. As a student, I observed with admiration his extraordinary capabilities of abstraction and presentation.

• Whenever I write a paper, or prepare a talk, I always use the Uri_Feige™ Latex/PowerPoint package.”

- Dan Vilenchik, BGU
Uri as an advisor

• “Working with Uri as an advisor was an inspiring experience, which helped me grow tremendously as a researcher.

• Privately, I used to call him "the oracle", for his tendency to spontaneously generate surprising insights and proof ideas almost mid-sentence, seemingly without any offline computational time.”

- Eden Chlamtac, BGU
Uri as an advisor

• “My main insight from Uri is to keep it simple and look for simple and elegant solutions. His ability to simplify complicated problems never stopped amazing me.

• To see Uri solve mathematical questions was similar to listen to Glenn Gould play Bach: everything is so accurate and crystal clear.”

- Daniel Reichman, WPI
What I learned: Be accurate, be modest

From: Uriel.Feige@weizmann.ac.il

• “In Section 1.4 and elsewhere there are claims of the form ‘will be of independent interest’.

• I recommend to write instead ‘may be of independent interest’…

• ...unless you know for sure that (a) it will be of interest, and (b) the interest will be independent of the application in the current paper.”
Beyond worst-case analysis in Uri’s Work

• Semi-random models:
  • A worst-case/average-case hybrid
  • Adversary and nature jointly produce problem instances

• [Feige-Krauthgamer’00, Feige-Kilian’01]:
  • Semi-random models for planted independent set
  • Insight into what properties of an IS make finding it easy

• Many additional works of Uri
  • Check out Uri’s forthcoming book chapter “Introduction to Semi-Random Models”
In this talk

• Some recent applications of the semi-random approach in algorithmic game theory (AGT)
  • [Carroll’17, Eden-Feldman-Friedler-T.C.-Weinberg’17, Duetting-Roughgarden-T.C.’19]

• A mystery in AGT:
  • Simple economic mechanisms are ubiquitous in practice...
  • ... but suboptimal in the worst-case and average-case sense
• Semi-random models help explain, quantify and improve
Intersection of disciplinary approaches

Algorithmic Game Theory

Algorithms  Microeconomics

Dominant approach: worst-case

Dominant approach: average-case
Mechanism design

Algorithm design with incentives, private information

- **Agents** use private information to maximize own utility
- Mechanisms use payments to maximize mechanism designer’s utility a.k.a. revenue
Auction and contract design

1. Auctions:
   • Agents are buyers (e.g., online advertisers)
   • Private info: Buyers’ values
   • Incentives: Auction induces buyers to bid their values

2. Contracts:
   • Agent hired to perform a task (e.g., online marketing)
   • Private info: Agent’s effort level
   • Incentives: Contract induces efficient effort level
Simple ubiquitous mechanisms

1. **Auctions:**
   - 2\textsuperscript{nd}-price auction – winner charged 2\textsuperscript{nd}-highest bid
   - No incentive to underbid
   - As seen on: eBay

2. **Contracts:**
   - Linear contract – agent gets a cut of her effort’s outcome
   - No incentive to slack off
   - As seen in: venture capital
Semi-random models for auctions

In what senses is the 2\textsuperscript{nd}-price auction optimal for multi-item revenue?
Multi-item auction setting

\[ n \text{ additive buyers} \]

\[ m = \Theta(n) \text{ items} \]
Bayesian (average-case) model

- Prior distributions $F_1, \ldots, F_m$ known to the auction
- Values sampled independently
- Auction gets bids, allocates items, charges payments

$n$ additive buyers

$m = \Theta(n)$ items
Average-case auction design?

• **Design problem**: Maximize expected revenue (total payment) subject to incentive compatibility (IC)
  • Expectation over priors $F_1, \ldots, F_m$
  • IC = true bids maximize buyer utilities

• **Notation**: $\text{OPT}_{F_1, \ldots, F_m}$

• Auctions achieving $\text{OPT}_{F_1, \ldots, F_m}$ unrealistically complex for $\geq 2$ items, and **brittle** even for 1 item
Worst-case auction design?

Nonstarter even for 1 item, 1 buyer with value $v$

- **Design problem**: Maximize revenue by setting reserve price $p$
- **But**: $\forall \ p \ \exists$ worst-case value $v$ s.t. revenue $= 0$
Semi-random to the rescue

• Semi-random models - recall:
  • A worst-case/average-case hybrid
  • Adversary and nature jointly produce problem instances

• In auctions:
  • Class of priors $\mathcal{F}$ known to auction
  • Adversary chooses worst-case prior $F \in \mathcal{F}$
  • Nature samples instance $\nu \sim F$
Semi-random instance generation

Class $\mathcal{F}$ of priors

Instance $\nu$ drawn from $F$
Two performance measures

Consider mechanism $M$

Recall $OPT_F = \mathbb{E}_F [\text{revenue of optimal mechanism for prior } F]$

1. **Relative**: $\min_{F \in \mathcal{F}} \left\{ \frac{\mathbb{E}_F [\text{revenue of } M]}{OPT_F} \right\}$

2. **Absolute**: $\min_{F \in \mathcal{F}} \{ \mathbb{E}_F [\text{revenue of } M] \}$
Two design goals

1. Maximize relative performance
   • Find $M$ that approximates $OPT_F$ simultaneously $\forall F \in \mathcal{F}$
   • Terminology: $M$ is prior-independent [Dhangwatnotai’15]

2. Maximize absolute performance
   • Find $M$ that achieves $\max \min \{\mathbb{E}_F[\text{revenue of } M']\}$
   • Terminology: $M$ is max-min optimal [Bertsimas’10, Carroll’19]

Choice of $\mathcal{F}$ is crucial
Recent results

• **Prior-independent auctions**
  1. Via extra buyers:
     • [Feldman-Friedler-Rubinstein EC’18] \((1 - \epsilon)\)-approximation
     • [Beyhaghi-Weinberg STOC’19] Improved and tight bounds
     • [Liu-Psomas SODA’18] Dynamic auctions
     • [Roughgarden-T.C.-Yan OR’19] Unit-demand buyers
  2. Via sampling + approximation:
     • [Allouah-Besbes EC’18] Lower bounds
     • [Babaioff-Gonczarowski-Mansour-Moran EC’18] Two samples
     • [Guo-Huang-Zhang STOC’19] Settling sample complexity

• **Max-min optimal auctions**
  • [Gravin-Lu SODA’18] With budgets
  • [Bei-Gravin-Lu-Tang SODA’19] Posted prices
Result 1: Max-min optimality [Carroll’17]

Setting: 1 buyer, $m$ items with priors $F_1, \ldots, F_m$

$\mathcal{F} =$ all correlated distributions with marginals $F_1, \ldots, F_m$

**Theorem [Carroll]:** Selling each item $j$ separately by 2nd-price auction with optimal reserve for $F_j$ is max-min optimal wrt $\mathcal{F}$

**Intuition:** Selling separately is robust to correlation
Max-min optimality

Distribution with marginals $F_1, \ldots, F_m$

Auction

Expected revenue

Min over columns

Max over rows
Robustness to correlation

Distribution with marginals $F_1, \ldots, F_m$

Setting separately

Same expected revenue
Towards result 2: What more do we want?

Recall theorem: Selling each item $j$ separately by 2\textsuperscript{nd}-price auction with optimal reserve for $F_j$ is max-min optimal wrt $\mathcal{F}$

Want: Prior-independence

- No reserve price tailored to $F_j$
- Revenue guarantee relative to $\text{OPT}_{F_1,\ldots,F_m}$

Willing to: assume values are independent
First attempt

**Setting**: \( n \) buyers, \( m \) items

\( \mathcal{F} = \) all product distributions \( F_1 \times \cdots \times F_m \) with regular marginals

**"Theorem"**: Selling each item \( j \) separately by 2\textsuperscript{nd}-price auction approximates \( \text{OPT}_{F_1,\ldots,F_m} \) simultaneously \( \forall F_1 \times \cdots \times F_m \in \mathcal{F} \)

**Counterexample**: 1 buyer
Resource augmentation

• Another beyond worst-case approach
• To compete with a powerful benchmark, the algorithm is allowed extra resources [Sleator-Tarjan’85]

• In our context [BulowKlemperer’96]:
  • Powerful benchmark is \( \text{OPT}_{F_1,...,F_m} \)
  • Resources are buyers competing for the items
Result 2: Prior-independence [Eden+’17]

**Theorem:** With $O(m)$ extra buyers, selling each item $j$ separately by 2nd-price auction matches $\text{OPT}_{F_1, \ldots, F_m}$ simultaneously $\forall F_1 \times \cdots \times F_m \in \mathcal{F}$
Result 2: Prior-independence [Eden+’17]

**Theorem:** With $O(m)$ extra buyers, selling each item $j$ separately by 2\textsuperscript{nd}-price auction matches $\text{OPT}_{F_1,\ldots,F_m}$ simultaneously $\forall F_1 \times \cdots \times F_m \in \mathcal{F}$

- [Feldman-Friedler-Rubinstein’18]: $\Omega(m)$ extra buyers necessary for $m = \Theta(n)$
- [Beyhaghi-Weinberg’19]: Additional tight results for other $n, m$ regimes
Auctions Recap

• For the canonical problem of maximizing revenue from $m$ items, semi-random models show that simple auctions are optimal.

  • Simple = selling each item by 2nd-price auction with reserve or more buyers.

  • Optimal =
    • Max-min optimal over adversarially chosen correlation or
    • Match $OPT_F$ simultaneously for any regular product distribution $F$. 
Semi-random models for contracts

In what sense are linear contracts optimal?
Bayesian model for contracts

• Agent has $n$ possible effort levels (hidden)
• Level $i$ induces a distribution over $m$ (observable) outcomes
  • $\mu_i$ = expected outcome
  • $c_i$ = cost
• Example:

<table>
<thead>
<tr>
<th></th>
<th>Low outcome $4$</th>
<th>Med. outcome $50$</th>
<th>High outcome $100$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low effort $0$</td>
<td>0.6</td>
<td>0.3</td>
<td>0.1</td>
</tr>
<tr>
<td>Med. effort $2$</td>
<td>0.4</td>
<td>0.4</td>
<td>0.2</td>
</tr>
<tr>
<td>High effort $9$</td>
<td>0.1</td>
<td>0.5</td>
<td>0.4</td>
</tr>
</tbody>
</table>

$\mu_1 = 27.4$
$\mu_2 = 41.6$
$\mu_3 = 65.4$
Bayesian model for contracts

• **Contract** = non-negative payment for every outcome
• **Revenue** = outcome minus payment
  • Measured in expectation over outcome distribution given effort

<table>
<thead>
<tr>
<th>Contract:</th>
<th>$2</th>
<th>$30</th>
<th>$45</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low effort $0</td>
<td>0.6</td>
<td>0.3</td>
<td>0.1</td>
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Linear contracts

• A linear contract is defined by a parameter $\alpha \leq 1$
• Agent chooses level $i^*$ that maximizes $\alpha \mu_i - c_i$
• Expected revenue is $(1 - \alpha)\mu_i^*$

<table>
<thead>
<tr>
<th>Contract:</th>
<th>$4\alpha =$2</th>
<th>$50\alpha =$25</th>
<th>$100\alpha =$50</th>
</tr>
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<tbody>
<tr>
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Result 3: Max-min optimality [Duetting+’19]

Setting: $n$ effort levels with expected outcomes $\mu_1, \ldots, \mu_n$

$\mathcal{F} =$ distributions $F_1, \ldots, F_n$ with expectations $\mu_1, \ldots, \mu_n$

**Theorem:** The optimal linear contract for $\mu_1, \ldots, \mu_n$ is max-min optimal wrt $\mathcal{F}$

See also [Carroll’15, Dai-Toikka’18, Carroll-Walton’20]
Same intuition as Result 1 for auctions

\[ F_1, \ldots, F_n \text{ with expectations } \mu_1, \ldots, \mu_n \]
Same intuition as Result 1 for auctions

\[ F_1, \ldots, F_n \text{ with expectations } \mu_1, \ldots, \mu_n \]
Adversary takes advantage of any non-robustness

Main lemma: ∀ contract, ∃ distributions with expectations $\mu_1, \ldots, \mu_n$ s.t. ∃ linear contract with better expected revenue
Take-away

Lots of recent beyond worst-case activity in AGT leading to new insights

“It is probably the great robustness of [simple mechanisms] that accounts for their popularity. That point is not made as effectively as we would like by our model; we suspect that it cannot be made effectively in any traditional Bayesian model.”

[Holmstrom-Milgrom’87]
Open problems

1. Auctions: beyond additive buyers?
2. Contracts: relative guarantees a la prior-independence?
3. General framework for max-min robustness?

Thanks for listening!